72311/A010

Reg. No. | | | | | | | | |

I Semester M.Sc. (CBCS) Degree Examination, April - 2021

MATHEMATICS

Algebra - I

Paper : 1.1

Time : 3 Hours **Maximum Marks : 80**

Instructions to Candidates :
1. Answer any five questions by choosing atleast **one** question from each part.
2. All questions carry **equal** marks.

PART - A

1. a) Prove that the set $G = \left\{ \begin{pmatrix} x & x \\ x & x \end{pmatrix} \Big/ x \neq 0, x \in R \right\}$ under matrix multiplication is a group. Is it abelian? Justify your answer. **(6+6+4)**

b) If G is a group in which $(ab)^m = a^m b^m$ for the consecutive integers, $\forall a, b \in G$, then prove that G is abelian.

c) If every element of G is its own inverse, then show that G is abelian.

2. a) Let G be a cyclic group generated by 'a' of order m. Then prove that 'a^k' generates G if and only if and only if $(k,m) = 1$. **(4+6+6)**

b) State and prove Lagrange's theorem for a finite group.

c) Let H and K be subgroup of G. Then prove that HK is a subgroup of G if and only if HK=KH.

3. a) Let G be group and N a normal subgroup of G. Then prove that $\frac{G}{N}$, collection of all right cosets of N in G, is a group. **(8+4+4)**

b) Show that the intersection of two normal subgroups If G is a normal subgroup of G.

c) Let ϕ be a homomorphism of G into $\overline{G}$. Then prove that

i. $\phi(e) = \bar{e}$, identity, element of $\overline{G}$.

ii. $\phi(x^{-1}) = [\phi(x)]^{-1}, \forall x \in G$

[P.T.O.

PART- B

4. a) State and prove fundamental theorem of homomorphism. **(8+8)**

   b) If N and M are normal subgroups of G, then prove that $\frac{NM}{M} \approx \frac{N}{N \cap M}$

5. a) Prove that a homomorphic image of a cyclic group is cyclic. **(4+8+4)**

   b) Prove that every group is isomorphic to a group of permutations.

   c) Prove that every permutation $\theta = I(identity)$ is a product of transposition.

6. a) Prove that the set A_n collection of all even permutation of n symbols, is a normal subgroup of S_n and $o(A_n) = \frac{n!}{2}$. **(8+8)**

   b) If G is a finite group and $H \neq G$ is a subgroup of G such that $o(G) + i_G(H)!$, then show that G cannot be simple.

PART - C

7. a) If $o(G) = p^n$, where P is a prime number, then prove that $Z \neq \{e\}$ where Z is the centre of a group G. **(8+8)**

   b) State and prove Sylow's theorem.

8. a) Suppose G is a finite abelian group and P/o(G), when P is a prime number. Then prove that there is an element $a \neq e$ in G of order P and G has a subgroup of order P. **(8+8)**

   b) Show that the Alternating group A_4 has no subgroup of order 6.